A Haplousterotic Model for the Nucleon Wave Function

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We review the consequences of the sensitivity of the relation between the *moments* of a model for the nucleon quark distribution amplitude and the *coefficients* of its polynomial expansion. Criteria for a simpler approach to constructing a model for the quark distribution amplitude are formulated. We describe how such a simpler (or *haplousterotic*) model for the quark distribution amplitude of the nucleon is obtained from the QCD sum-rule moments of COZ.

1 Introduction

Although the knowledge of the nucleon wave function in terms of its fundamental quark and gluon degrees of freedom is of outstanding theoretical interest because of its *process-independence*, there are many drastically different model amplitudes for the nucleon available as shown in Fig. 1. All the functions shown are intended to describe the distribution of the longitudinal momentum fractions of the quarks inside the nucleon.

In calculations using perturbative Quantum Chromodynamics (pQCD), not the full wave function ψ_N itself is used, but the quark distribution amplitude (QDA) ϕ_N, which is defined from the nucleon wave function by

$$\phi_N(x, \mu^2) := \int_{\vec{k_\perp} < \mu^2} [d^2 k_{\perp i}] \; \psi_N(x_i, \vec{k_{\perp i}}) \quad . \tag{1}$$

From Eq. 1 is clear that ϕ_N is also process-independent and can thus be used in calculations of various observables such as formfactors, decay widths or amplitudes in virtual Compton scattering[8] (VCS). In the hard-scattering aproach[9] of pQCD the contributions to such processes are assumed to factorize into a process-specific hard-scattering amplitude T_H between the distribution amplitudes of in- and outgoing states. For example, a typical contribution to T_H in VCS looks like the diagram in Fig. 2.

Whereas the hard-scattering amplitude can be calculated perturbatively by exchanging hard gluons, the QDA ϕ_N is a purely non-perturbative object and must be obtained from elsewhere.

Usually, the ansatz for ϕ_N was an expansion into a set of orthogonal polynomials (Appell polynomials $A_i(x_1, x_2, x_3)$ for the nucleon[9,10]). The remaining

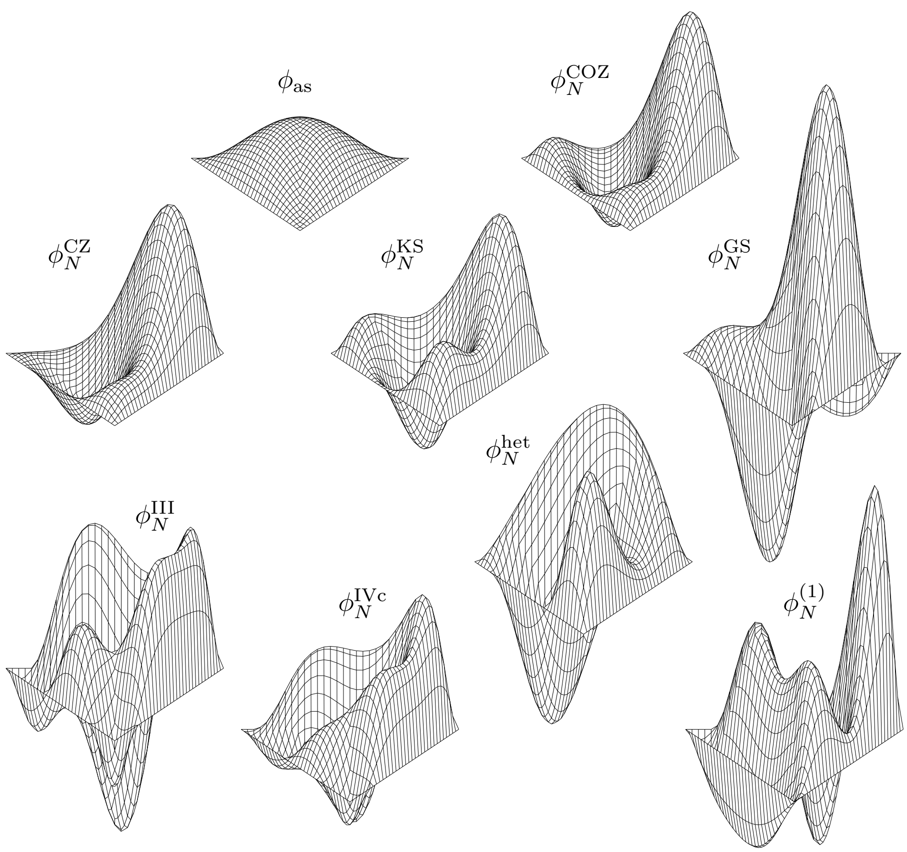

Figure 1: Some selected models for the nucleon wave function. The model amplitudes are polynomial approximations of second and third degree from Refs. 1 (COZ), 2 (CZ), 3 (KS), 4 (GS), 5 (het), 6 (III, IVc), and 7 (1). $\phi_{\rm as} = 120\, x_1 x_2 x_3$ denotes the asymptotic form.

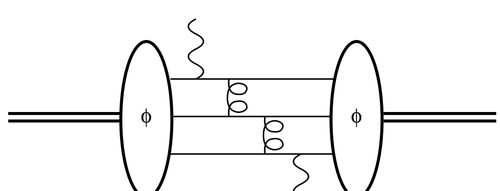

Figure 2: A typical lowest-order contribution to virtual compton scattering in the hard-scattering picture of pQCD. While the hard gluon exchanges can be calculated perturbatively, the nucleon QDA ϕ that contains all the non-perturbative information is usually obtained from QCD sum-rules.

task was to find the corresponding expansion coefficients c_i. From the definition of the moments of ϕ_N

$$\begin{aligned}\langle ijk\rangle := \langle x_1^i x_2^j x_3^k\rangle \quad &:= \int [dx]\ x_1^i\ x_2^j\ x_3^k \cdot \phi_N(x_1,x_2,x_3) \\ &= \sum_{i=0}^{\infty} c_i \cdot \int [dx]\ x_1^i\ x_2^j\ x_3^k \cdot A_i(x_1,x_2,x_3)\ ,\end{aligned} \tag{2}$$

which weight the QDA with different powers of the x_i, it is clear that there exists a *linear relation* between the *moments* of the QDA and its *expansion coefficients* c_i. It is thus obvious that a simple matrix inversion yields the coefficients c_i as linear functions of the moments of ϕ_N. If the moments were known exactly for all orders, this would allow to reconstruct the QDA precisely.

However, the non-perturbative methods presently available for actual calculations, such as QCD sum rules or QCD lattice gauge theory, can only provide the lowest order moments up to a quite limited accuracy of up to 30%.

Furthermore, as was shown in previous works[11], the relation is extremely sensitive to uncertainties or errors in the moments. This is so because the matrix which has to be inverted, has a *nearly* vanishing determinant[11] or – equivalently – some very small eigenvalues[3]. The problem is *ill-posed*.

The uncertainties, which can be as large as 30%, destroy the information about the coefficients, e.g. by a sign flip. Therefore, the model QDAs not only show very different shapes for the same range of the moments input, but in addition the polynomial expansion does not even converge. Thus, there is an increase of the oscillations with increasing degree of the expansion, instead of an expected gradual refinement.

So what can be done in order to circumvent the above difficulties ?

2 A simpler approach

In order to obtain a more reasonable model of the QDA which avoids the problems of unphysical oscillations (due to a failure in the fine-tuning of higher-order expansion coefficients), we list some criteria of simplicity that a physical distribution amplitude should fulfill *in addition to QCD sum rules*[12]:

1. functional simplicity (e.g. an exponential ansatz)
2. minimum number of parameters
3. smooth

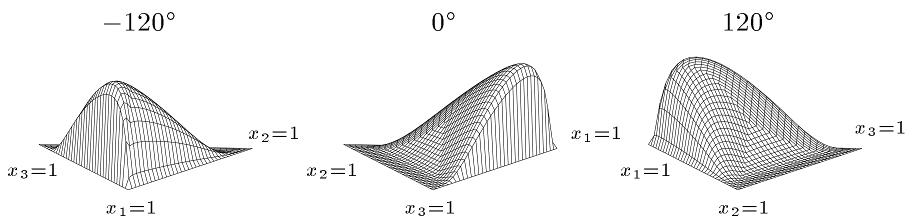

Figure 3: The haplousterotic model for the nucleon quark distribution amplitude shown from different directions. This model amplitude is of exponential type – similar to a model used in Ref. 13 – and was determined using the QCD sum-rule moments up to third order of Ref. 1. It reaches the edge everywhere with slope 0, although this may not be evident from the figure.

4. no oscillations
5. positive
6. substantially non-polynomial
7. no specific process (experiment) as input (process-independence)

A model constructed in such a way will be called "haplousterotic" (from the greek word $\alpha\pi\lambda o\upsilon\sigma\tau\varepsilon\rho o\varsigma$ for "simpler").

Using the above criteria, our haplousterotic model amplitude $\phi_N^{\mathrm{Ha}+}$ was determined from the QCD sum-rule moments of COZ[1]. The model is shown in Fig. 3. It has the form[12]

$$\phi_N^{\mathrm{Ha}+}(x) = N \exp\left(-(\frac{b_1^{(r)}}{x_1^r} + \frac{b_2^{(r)}}{x_2^r} + \frac{b_3^{(r)}}{x_3^r})\right) \quad , \tag{3}$$

with $N = 14.626$, $b_1^{(1)} = 0.5880$, $b_2^{(1)} = 8.724 \times 10^{-13}$ and $b_3^{(1)} = 0.02413$ for $r = 1$.[a]

Of course, for the purpose of investigation of the Q^2-evolution of the QDA or its convergence properties, $\phi_N^{\mathrm{Ha}+}$ can be expanded into a series of Appell polynomials. A graphical representation of different orders of expansion is shown in Fig. 4. As it should be, one can observe a nice convergence. But one also has to notice that very high polynomial degrees are needed for the expansion to resemble the shape of the exact model $\phi_N^{\mathrm{Ha}+}$. On the other hand, it is interesting to note that the second degree approximation to $\phi_N^{\mathrm{Ha}+}$ looks very much like the early model of CZ (cf. Fig. 1).

[a] There was a factor 3 missing in the $b_i^{(r)}$ in the first reference of Ref. 12.

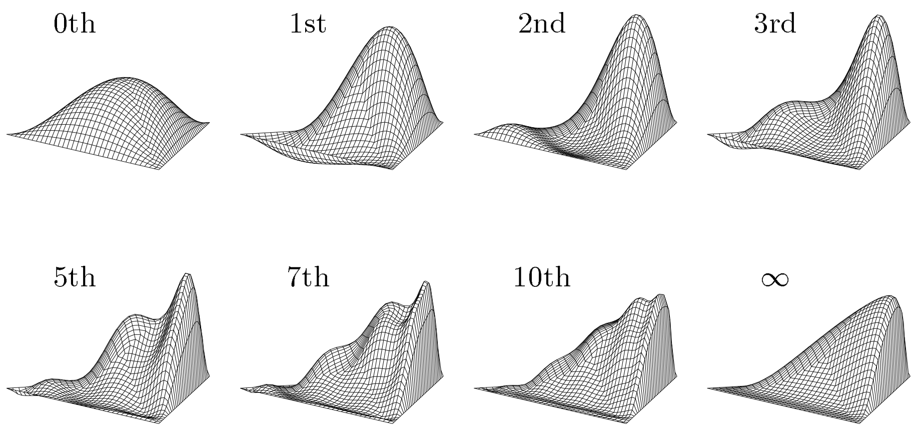

Figure 4: The haplousterotic model amplitude is expanded into Appell polynomials. However, to resemble the shape of the exact (∞) form, polynomials of very high degree are needed.

Finally, we would like to emphasize that a fixed model amplitude of the nucleon can be used to study systematically the effects of various uncertainties and approximations that are necessary to apply perturbative QCD in the calculation of physically relevant quantities like form factors, decay widths etc.. Relevant problems in this context are e.g. constant or dynamical α_S[7] and related end-point problems[14], choice of Λ_{QCD}, $k_\perp$-[7,15] and Sukakov[16,17] effects, $k_\perp$-dependence of the full wave function, higher-twist effects, higher-order α_S etc.

3 Summary

Due to the sensitivity of the relation between moments and coefficients to small variations in the moments (which are unavoidable when the moments are calculated, e.g. by the method of QCD sum rules) strong oscillations result in the polynomial approximation, which lead to a non-convergence.

A way out of this dilemma is to restrict the structure of the quark distribution amplitude to physical and phenomenological criteria of simplicity unless significantly more accurate methods for obtaining QCD wave functions are available.

Our haplousterotic model of the nucleon QDA contains all available and physically relevant information and it will be an interesting challenge to test it in VCS.

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